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Numerical Predictions of Ship Motions at High Forward Speed [and Discussion]

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Numerical predictions of ship motions at high forward speed

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A theory for ship motions at high forward speed is presented. The theory includes interaction between the steady and unsteady flow field. Numerical results for the steady flow and added mass and damping are compared with experimental results.

1. Introduction

Generally speaking strip theories are still the most successful theories for wave-induced motions of ships at moderate forward speed. However, from a theoretical point of view one can question strip theories. For instance, strip theories account for the interaction with the forward speed in a simplistic way. The effect of the local steady flow around the ship is neglected. Further the classical linear free surface condition with forward speed is simplified so that the unsteady waves generated by the body are propagating in directions perpendicular to the longitudinal axis of the ship. If the complete classical linear unsteady free surface condition with forward speed were used, a more complex wave system would have appeared.

Strip theories have no justification for high speed. Chapman (1975) has presented a simplified high-speed theory for a vertical surface-piercing flat plate in unsteady yaw and sway motion. In the main text we present a generalization of Chapman's method to analysis of any type of slender high-speed ship in waves. The steady flow problem is nonlinear, while the unsteady flow is based on linear theory. Interaction between the local steady flow and the unsteady flow is accounted for. In developing the theory we have had in mind that it should be practical. This rules out a complete three-dimensional theory.

2. Theory

Consider a slender ship at high Froude number in incident regular waves on deep water. A right-handed coordinate system (x, y, z) fixed with respect to the mean oscillatory position of the ship is used, with positive z vertically upwards through the centre of gravity of the ship and the origin in the plane of the undisturbed free surface. The ship is assumed to have the xz plane as a plane of symmetry in its mean oscillatory position. Let the translatory displacements in the x-, y- and z-directions with respect to the origin be η_1, η_2 and η_3, respectively, so that η_1 is the surge, η_2 is the sway and η_3 is the heave displacement. Furthermore, let the angular displacement of the rotational motion about the x-, y- and z-axes be η_4, η_5 and η_6, respectively, so that η_4 is the roll, η_5 is the pitch and η_6 is the yaw angle. The coordinate system and the translatory and angular displacement conventions are shown in figure 1.

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Figure 1

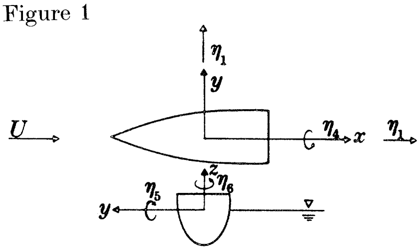

Figure 2

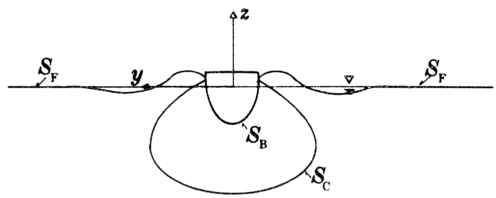

Figure 1. Coordinate system and definitions of translatory and angular displacements of a ship.
Figure 2. Control surfaces used in the solution of the forced motion problem.

The problem can be formulated in terms of potential flow theory. The unsteady motions of the ship and the fluid is assumed to be small so that we can linearize the unsteady body boundary and free surface conditions. We assume that steady-state conditions have been obtained and write the total velocity potential as

$$\Phi = Ux + \phi_s(x,y,z) + \phi_1 e^{i\omega t}, \tag{2.1}$$

where U is the forward speed of the ship, ω is the frequency of encounter between the waves and the ship, t is the time variable and i the imaginary unit. It is understood that real parts should be taken of the time-dependent part of equation (2.1).

The boundary value problem will be simplified by introducing the slenderness parameter ϵ. This expresses the order of magnitude of the ratio of the beam or draught of the ship length. We assume that $\partial f/\partial x = O(f\epsilon^{-\frac{1}{2}})$, $\partial f/\partial y = O(f\epsilon^{-1})$, $\partial f/\partial z = O(f\epsilon^{-1})$ where f is any flow variable caused by the body in some region near the ship. Further $n_1 = O(\epsilon^{\frac{1}{2}})$ where n_1 is the x-component of a unit normal vector $\boldsymbol{n}$ to the wetted part of the ship surface. Positive normal direction is into the fluid domain. A motivation for choosing $n_1 = O(\epsilon^{\frac{1}{2}})$ is that n_1 is often relatively large in the bow and stern region and quite small along the midbody part. By setting $n_1 = O(\epsilon^{\frac{1}{2}})$ we imply that the bow and stern region is $O(\epsilon^{\frac{1}{2}})$. By setting $\partial f/\partial x = O(f\epsilon^{-\frac{1}{2}})$ we are allowing for variation of the flow in the x-direction over a length scale $O(\epsilon^{\frac{1}{2}})$. The reason why we have chosen $\epsilon^{\frac{1}{2}}$ as a length scale and not any other value ϵ^a $(0 \leqslant a \leqslant 1$, $a \neq \frac{1}{2})$ is that $\epsilon^{\frac{1}{2}}$ leads to a meaningful set of equations and boundary conditions. U or Froude number based on the ship length are assumed to be $O(1)$.

Steady flow problem

It can be shown that the steady flow velocity potential ϕ_s must satisfy

$$\partial^2\phi_s/\partial y^2 + \partial^2\phi_s/\partial z^2 = 0 \quad \text{in the fluid domain,} \tag{2.2}$$

$$\frac{\partial \phi_s}{\partial x} = -\frac{g}{U}\zeta_s - \frac{1}{2U}\left[\left(\frac{\partial\phi_s}{\partial y}\right)^2 + \left(\frac{\partial\phi_s}{\partial z}\right)^2\right] \quad \text{on } z = \zeta_s(x,y), \tag{2.3}$$

$$\frac{\partial \zeta_s}{\partial x} = -\frac{1}{U}\frac{\partial\phi_s}{\partial z} - \frac{1}{U}\frac{\partial\phi_s}{\partial y}\frac{\partial\zeta_s}{\partial y} \quad \text{on } z = \zeta_s(x,y). \tag{2.4}$$

The terms that we have neglected are $O(\epsilon)$ relative to the terms that we have kept.

The fluid domain is defined to be outside and near the mean oscillatory position of the ship; $\zeta_s(x, y)$ means the steady free surface elevation. The following body boundary condition applies

$$\partial\phi_s/\partial N = -Un_1, \tag{2.5}$$

where $\partial/\partial N$ means derivative along a unit vector $\boldsymbol{N}$ which is perpendicular to the contour of the body in a cross-plane. Positive direction of $\boldsymbol{N}$ is into the fluid domain. To solve equations (2.2) to (2.5) it is necessary to set starting conditions at the bow $x = x_B$. We set the velocity potential and free-surface elevation equal to zero at $x = x_B$, that is

$$\phi_s = 0 \quad \text{and} \quad \zeta_s = 0 \quad \text{at} \quad x = x_B. \tag{2.6}$$

A solution can be found by starting at the bow, use equations (2.3) and (2.4) to step the solution of the free surface elevation ζ_s and the velocity potential ϕ_s on $z = \zeta_s$ in the x-direction. For each cross section we can represent the velocity potential at a point (y, z) as

$$-2\pi\phi_s(y,z) = \int_{S_F,\,S_B} \left[\phi_s(\eta,\zeta)\frac{\partial \ln r}{\partial N(\eta,\zeta)} - \ln r\frac{\partial\phi_s(\eta,\zeta)}{\partial N(\eta,\zeta)}\right] \mathrm{d}s(\eta,\zeta). \tag{2.7}$$

Here $r = [(y-\eta)^2+(z-\zeta)^2]^{\frac{1}{2}}$, S_B the wetted body surface, S_F the exact free surface, $\mathrm{d}s$ a surface element along either S_F or S_B and $\partial/\partial N$ is the derivative along the perpendicular to either S_B or S_F in the cross-plane. Positive normal direction is into the fluid domain. Since there are no waves far away from the ship, the contribution from the free-surface integral part of equation (2.7) can be rewritten. For $|y| > b(x)$, where $b(x)$ is large relative to the cross-dimensions of the ship, we can write

$$\phi_s(y,z) \sim Az/(y^2+z^2), \tag{2.8}$$

where A is a constant depending on x. This means ϕ_s has a vertical dipole behaviour far away from the ship. Unknowns in equation (2.7) are ϕ_s on the body surface and $\partial\phi_s/\partial N$ on the free surface. These are found by solving the integral equation that arises when (y, z) approaches points on S_F and S_B.

From equation (2.5) we see that $\phi_s = O(\epsilon^{\frac{3}{2}})$ and from equation (2.3) we find that the steady wave elevation is the order of magnitude of the transverse dimensions of the ship. This is the reason why we cannot linearize the free surface condition about $z = 0$.

In the derivation of the boundary value problem we have said that the Froude number is order 1 and that $\partial\phi_s/\partial x = O(\phi_s\,\epsilon^{-\frac{1}{2}})$. We can get a feeling of what the solution represents if we linearize the problem and represent the far-field flow created in the bow by a source. Ogilvie (1977) has actually shown that the linear solution represents the diverging waves caused by the ship. The effect of the transverse waves are neglected. The solution is appropriate for the bow flow of any fine ship form at any Froude number. The higher the Froude number is, the longer is the distance along the ship where the solution is appropriate.

Forced motion problem

In the unsteady flow problem we concentrate on the forced motion problems. There are no incident waves. That means we write

$$\phi_1\,\mathrm{e}^{\mathrm{i}\omega t} = \sum_{j=2}^{6} \Phi_j\,\eta_j, \tag{2.9}$$

where the effect of surge is higher order. It can be shown that Φ_j must satisfy

$$\partial^2\Phi_j/\partial y^2+\partial^2\Phi_j/\partial z^2 = 0 \quad \text{in the fluid domain,} \tag{2.10}$$

$$\frac{\partial\Phi_j}{\partial x} = -\frac{i\omega}{U}\Phi_j-\frac{\zeta_j}{U}\left[g+U\frac{\partial^2\phi_s}{\partial z\,\partial x}+\frac{\partial\phi_s}{\partial y}\frac{\partial^2\phi_s}{\partial z\,\partial y}+\frac{\partial\phi_s}{\partial z}\frac{\partial^2\phi_s}{\partial z^2}\right]$$
$$-\frac{1}{U}\left[\frac{\partial\phi_s}{\partial y}\frac{\partial\Phi_j}{\partial y}+\frac{\partial\phi_s}{\partial z}\frac{\partial\Phi_j}{\partial z}\right] \quad \text{on } z=\zeta_s(x,y), \tag{2.11}$$

$$\frac{\partial\zeta_j}{\partial x} = \frac{\zeta_j}{U}\left[-i\omega+\frac{\partial^2\phi_s}{\partial z^2}-\frac{\partial^2\phi_s}{\partial y\,\partial z}\frac{\partial\zeta_s}{\partial y}\right]$$
$$+\frac{1}{U}\frac{\partial\Phi_j}{\partial z}-\frac{1}{U}\left[\frac{\partial\zeta_s}{\partial y}\frac{\partial\Phi_j}{\partial y}+\frac{\partial\phi_s}{\partial y}\frac{\partial\zeta_j}{\partial y}\right] \quad \text{on } z=\zeta_s(x,y). \tag{2.12}$$

The terms involving second derivatives of ϕ_s are obtained by a Taylor expansion about $z=\zeta_s(x,y)$. This requires certain regularity conditions which may not always be satisfied at the intersection between the free surface and the body surface. This may represent problems if the ship is not vertically wall-sided at the free surface.

The fluid domain is defined to be outside and near the mean oscillatory position of the ship, $\zeta_j\,\eta_j$ means the unsteady free surface elevation relative to the steady free surface elevation ζ_s. We have not yet said anything about the order of magnitude of ω. For the terms involving ω in equations (2.11) and (2.12) to be of equal importance with the rest of the terms, it is necessary that $\omega=O(\epsilon^{-\frac{1}{2}})$. If $\omega=o(\epsilon^{-\frac{1}{2}})$, we can still keep the ω-terms as long as we only claim that we solve the problem to the order of magnitude of the terms that do not involve ω. The terms that we have neglected in equations (2.10) to (2.12) are $O(\epsilon)$ relative to terms that we have kept. The following body boundary condition applies

$$\partial\Phi_j/\partial N = i\omega n_j+m_j \tag{2.13}$$

on the mean oscillatory position of the ship. The components n_j and m_j are defined by

$$\left.\begin{aligned}&(n_1,n_2,n_3)=\boldsymbol{n},\ (n_4,n_5,n_6)=(yn_3-zn_2,\ -xn_3,\ xn_2),\\ &m_2=-\frac{\partial}{\partial N}\frac{\partial\phi_s}{\partial y},\quad m_3=-\frac{\partial}{\partial N}\frac{\partial\phi_s}{\partial z},\quad m_4=\frac{\partial\phi_s}{\partial y}n_3-\frac{\partial\phi_s}{\partial z}n_2+ym_3-zm_2,\\ &m_5=-Un_3-xm_3,\quad m_6=Un_2+xm_2.\end{aligned}\right\} \tag{2.14}$$

The term proportional to U in m_5 and m_6 is $O(\epsilon^{\frac{1}{2}})$ relative to the other term in m_5 or m_6. The terms involving second derivatives of ϕ_s are obtained by a Taylor expansion about the mean oscillatory position of the body surface. This may represent problems at sharp corners on the body surface and at the intersection between the free surface and the body surface.

We note there are interactions with the local steady flow both in the body boundary and free surface conditions. In the body boundary conditions the interaction terms occur because the steady flow satisfy the body boundary conditions on the mean oscillatory position and not the instantaneous position of the ship. If the interaction with ϕ_s is neglected in the free surface conditions, we obtain the classical free surface condition applicable to, for instance, a harmonically oscillatory source with mean forward speed. Sometimes the classical free surface condition is combined with the presence of ϕ_s in the m_j-terms of the body boundary condition. Our analysis shows that the ϕ_s-terms should be kept both in the body boundary and free surface conditions.

To solve equations (2.11) to (2.13) it is necessary with starting conditions at the bow. This is done similarly as in the steady flow problem (see equation (2.6)).

A solution can be found by starting at the bow and use equations (2.11) and (2.12) to step the solution of the unsteady free surface position ζ_j and velocity potential Φ_j on $z = \zeta_s$ in the x-direction. For each cross section we represent the velocity potential in somewhat different form than in the steady flow problem. The reason for doing this is to avoid evaluating the second derivatives of ϕ_s in the body boundary conditions. This causes numerical problems in particular near sharp corners.

We will introduce particular solutions Φ_{pj} that satisfy two-dimensional Laplace equation in cross-planes of the ship and the terms involving ϕ_s in the body boundary conditions for Φ_j. We can write

$$\left.\begin{aligned} &\Phi_{p2} = -\partial\phi_s/\partial y, \quad \Phi_{p3} = -\partial\phi_s/\partial z, \quad \Phi_{p4} = -y\,\partial\phi_s/\partial z + z\,\partial\phi_s/\partial y, \\ &\Phi_{p5} = x\,\partial\phi_s/\partial z, \quad \Phi_{p6} = -x\,\partial\phi_s/\partial y. \end{aligned}\right\} \tag{2.15}$$

From Green's second identity we can write

$$2\pi\Phi_j(y,z) = \int_{S_B+S_F}\left(\ln r\frac{\partial\Phi_j}{\partial N} - \Phi_j\frac{\partial}{\partial N}\ln r\right)\mathrm{d}s(\eta,\zeta), \tag{2.16}$$

$$2\pi C\Phi_{pj}(y,z) = \int_{S_B+S_c}\left(\ln r\frac{\partial\Phi_{pj}}{\partial N} - \Phi_{pj}\frac{\partial}{\partial N}\ln r\right)\mathrm{d}s(\eta,\zeta), \tag{2.17}$$

where $r = [(y-\eta)^2+(z-\zeta)^2]^{\frac{1}{2}}$. $C = 1$ when (y,z) is inside S_B+S_c and equal to zero when (y,z) is outside S_B+S_c. S_B, S_F and $\mathrm{d}s$ is defined in connection with equation (2.7). S_c is illustrated in figure 2 and is selected so that S_B+S_c encloses a fluid volume. Otherwise we are free how to select S_c. From a computational point of view it is important that S_c does not contain parts with a high curvature. The procedure does not solve possible numerical problems with second derivatives of ϕ_s at the intersection between the free surface and the body surface. By subtracting equations (2.16) and (2.17) we can write

$$\begin{aligned} 2\pi(\Phi_j - C\Phi_{pj}) = &\int_{S_B}\left[\ln r\frac{\partial}{\partial N}(\Phi_j-\Phi_{pj}) - (\Phi_j-\Phi_{pj})\frac{\partial}{\partial N}\ln r\right]\mathrm{d}s \\ &+\int_{S_F}\left[\ln r\frac{\partial}{\partial N}\Phi_j - \Phi_j\frac{\partial}{\partial N}\ln r\right]\mathrm{d}s - \int_{S_c}\left[\ln r\frac{\partial}{\partial N}\Phi_{pj} - \Phi_{pj}\frac{\partial}{\partial N}\ln r\right]\mathrm{d}s. \end{aligned} \tag{2.18}$$

In the integral over S_B we can write $(\partial/\partial N)(\Phi_j - \Phi_{pj}) = \mathrm{i}\omega n_j + M_j$, where $M_j = 0$, $(j = 2, 4)$, $M_5 = -Un_3$, $M_6 = Un_2$. What we have obtained by writing the solution in the form of equation (2.18) is that it is unnecessary to evaluate second derivatives of ϕ_s on S_B. Instead we have to evaluate them along S_c, where we do not have the same numerical difficulties as along S_B except at the intersection between the hull surface and the free surface.

The integration along S_F in equation (2.18) can be simplified in the same way as in the steady flow problem. When $|y| > b_j(x)$ we can write

$$\Phi_j \approx \begin{cases} A_j z/(y^2+z^2), & j = 3, 5, \\ A_j zy/(y^2+z^2)^2, & j = 2, 4, 6. \end{cases} \tag{2.19}$$

Unknowns in equation (2.18) are $\Phi_j - \Phi_{pj}$ on the body surface and $\partial\Phi_j/\partial N$ on the free surface. They are found by setting up an integral equation in the same way as in the steady flow problem.

If the hull ends in a trailing edge, we assume in the sway, roll and yaw problems that there is a vortex sheet leaving from the trailing edge in the downstream direction. This means we consider the ship to be a low-aspect-ratio lifting surface. The upstream effect of the vortex sheet is negligible.

If we neglect the effect of ϕ_s in the problem, it can be shown that the solution accounts for the unsteady diverging waves only (Faltinsen 1983). In the heave and pitch problem there are unsteady transverse wave systems that are neglected. This is appropriate for high Froude numbers. Since there are no upstream effects in the solution, it is necessary that $\tau = \omega U/g > \frac{1}{4}$.

Added mass, damping and restoring coefficients

The added mass, damping and restoring coefficients in the equations of motions for the ship oscillations can be found by starting out with Bernoulli's equation for the pressure and neglecting higher-order terms. We can write the pressure as

$$p = -\rho g z - \rho U \frac{\partial \phi_s}{\partial x} - \tfrac{1}{2}\rho\left[\left(\frac{\partial \phi_s}{\partial y}\right)^2 + \left(\frac{\partial \phi_s}{\partial z}\right)^2\right] - \rho g(\eta_3 + y\eta_4 - x\eta_5)$$

$$-\rho \sum_{j=2}^{6}\left[\mathrm{i}\omega\Phi_j + U\frac{\partial \psi_j}{\partial x} + \frac{\partial \phi_s}{\partial y}\frac{\partial \psi_j}{\partial y} + \frac{\partial \phi_s}{\partial z}\frac{\partial \psi_j}{\partial z}\right]\eta_j - \rho U \frac{\partial \phi_s}{\partial z}\eta_5 + \rho U \frac{\partial \phi_s}{\partial y}\eta_6, \tag{2.20}$$

where $\psi_j = \Phi_j - \Phi_{pj}$. The three first terms are steady. All terms should be evaluated at the mean oscillatory position of the ship. In the last terms we should note the presence of the ψ_j-terms. They are a combined effect of the unsteady Φ_j-potential and an effect of that the ship oscillates in its local steady flow. The last effect is common to neglect in ship motion theories. Keuning (1988) defined the latter effect by restoring coefficients in his experimental studies of heave added mass and damping of a high-speed ship. He obtained the terms by displacing the ship vertically both upwards and downwards relative to the reference position. By measuring the vertical steady force at each position, he derived a heave restoring coefficient. Both the heave velocity and acceleration were zero during these experiments. This corresponds to setting $\omega = 0$ in our analysis. However, when we do the calculations for finite ω-value, the interaction with the local steady flow is not the same as for $\omega = 0$.

The force and moments on the ship can be written as

$$F_k = -\int_{S_B} p n_k \, \mathrm{d}s \quad (k = 1, \ldots, 6),$$

where $\boldsymbol{F} = (F_1, F_2, F_3)$ is the force and $\boldsymbol{M} = (F_4, F_5, F_6)$ is the moment acting on the ship. S_B is the wetted body surface. This includes the steady wave elevation along the ship. In the case of no motions F_1 is the wave resistance of the ship. F_3 and F_5 can be used to find the steady 'sinkage' and trim of the ship. We concentrate on the linear unsteady forces. The contribution from the fourth term in equation (2.20) can be evaluated by Gauss theorem. The resulting force and moment components will be expressed by restoring terms $-C_{kj}\eta_j$. By adding the effect of the mass of the ship, we can write

$$\left.\begin{aligned} &C_{33} = \rho g A_{\text{wp}}, \quad C_{35} = C_{53} = -\rho g \int_{A_{\text{wp}}} x\,\text{d}s, \quad C_{55} = \rho g \int_{A_{\text{wp}}} x^2\,\text{d}s, \\ &C_{44} = \rho g \int_{A_{\text{wp}}} y^2\,\text{d}s + \rho g V z_{\text{B}} - M g z_{\text{G}}. \end{aligned}\right\} \tag{2.21}$$

The other C_{jk}-terms are zero. Here A_{wp} is the water plane area of the ship defined by the steady wave elevation along the ship (local spray is disregarded). V is the submerged volume of the ship relative to the steady wave elevation, z_{B} is the z-coordinate of the centre of buoyancy of V, M is the mass of the ship, g is acceleration of gravity and z_{G} is the z-coordinate of the centre of gravity of ship. We should note that A_{wp}, V and z_{B} have different meaning in conventional ship motion theories. A_{wp} is for instance the water plane area in calm water.

The contribution from the last terms in equation (2.20) is written in the form of added mass (A_{kj}) and damping coefficients (B_{kj}). There is an ambiguity in what we define as restoring and added mass coefficients. Any restoring coefficient can be written as an added mass coefficient and vice versa. In the definition of added mass it is important to know what we define as restoring coefficient. We can write

$$A_{kj} = \text{Re}\,(T_{kj})/\omega^2, \quad B_{kj} = -\text{Im}\,(T_{kj})/\omega, \tag{2.22}$$

where

$$T_{kj} = \rho \int_{S_{\text{B}}} n_k \left[\text{i}\omega \Phi_j + U \frac{\partial \psi_j}{\partial x} + \frac{\partial \phi_{\text{s}}}{\partial y}\frac{\partial \psi_j}{\partial y} + \frac{\partial \phi_{\text{s}}}{\partial z}\frac{\partial \psi_j}{\partial z} \right] + G_{kj}. \tag{2.23}$$

G_{kj} is zero except for

$$G_{k5} = \rho U \int_{S_{\text{B}}} \frac{\partial \phi_{\text{s}}}{\partial z} n_k\,\text{d}s \qquad (k = 3, 5),$$

$$G_{k6} = -\rho U \int_{S_{\text{B}}} \frac{\partial \phi_{\text{s}}}{\partial y} n_k\,\text{d}s \quad (k = 2, 4, 6).$$

3. Numerical solution and validation

In the numerical solution based on equations (2.7) and (2.18) the control surfaces are divided into a number of segments, and the velocity potentials and their normal derivatives are set constant over each segments. Only segments for $|y| < b(x)$ (or $b_j(x)$) are used on the free surface. The integral equations resulting from (2.7) and (2.18) are satisfied on the midpoint of each element.

The expressions for the velocity potential and free surface elevations on the free surface was obtained by equations (2.3), (2.4), (2.11) and (2.12). This was done by a second-order Runge–Kutta method. Numerical results have been compared with the results given by Faltinsen (1983) for linear bow flows and transient motion problems. Good agreement was documented.

We have also tested our numerical results against Tuck's (1988) numerical results

Figure 3

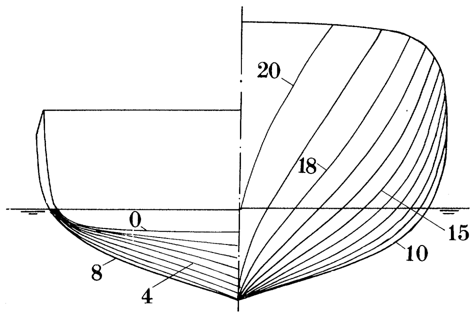

Figure 4

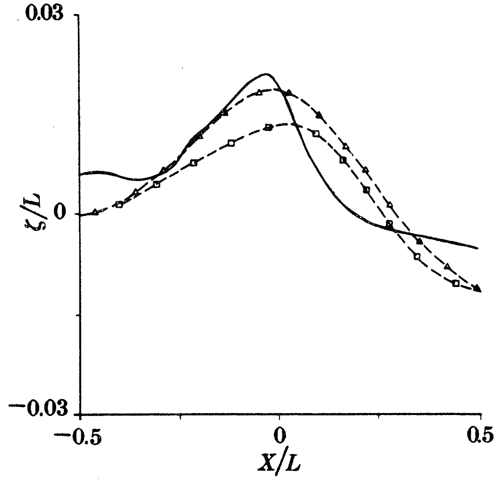

Figure 3. Body plan of the model used in Keuning's experiments (1988).

Figure 4. Measured and computed wave profile along the length of the model presented in figure 3 and table 1. ——, Experiments, Keuning (1988); —△—, nonlinear theory; —□—, linear theory. ζ is the wave elevation, L is the ship length, X is the longitudinal coordinate, $X = -\frac{1}{2}L$ at the bow. $Fn = 1.14$. Trim is 1.62°.

Table 1. *Main particulars of the model used in Keuning's experiments (1988)*

length of test waterline	2.00 m	draft	0.0624 m
beam of test waterline	0.25 m	block coefficient	0.396

of a wave resistance of a parabolic strut. Tuck used both Michell integral and what he calls a strip theory. In the strip theory he linearizes the free-surface conditions and satisfies the body boundary condition on the centre plane of the ship. Otherwise the boundary value problem is similar as in our approach. We find good agreement with Tuck's strip theory when we linearize the free-surface conditions. There is good agreement with the more exact Michell integral at high Froude number (greater than *ca.* 0.5). However, for small Froude numbers the disagreement is large. A reason is that we neglect the effect of the transverse wave systems.

Our numerical results were also compared with the numerical and experimental results presented by Chapman (1975) for a flat plate oscillating in sway and moving with forward speed. In this case the steady velocity potential ϕ_s is zero and our theory is identical to Chapman's theory. We found good agreement with Chapman's result. Chapman shows satisfactory results relative to model tests for Froude numbers as low as 0.16. One reason is probably that neglection of the transverse wave systems does not influence the sway-problem. Chapman also used the strip theory by Salvesen *et al.* (1970). The results by strip theory were unsatisfactory for the higher Froude numbers. The highest Froude number was 0.96.

Keuning (1988) studied experimentally the distribution of added mass and damping in heave along a high speed displacement hull. The body plan of the model is shown in figure 3. The main parameters of the ship model are given in table 1. Froude numbers 0.57 and 1.14 were examined. Since the ship hull is not vertically wall-sided at the free surface, it represents a difficult validation test of our numerical

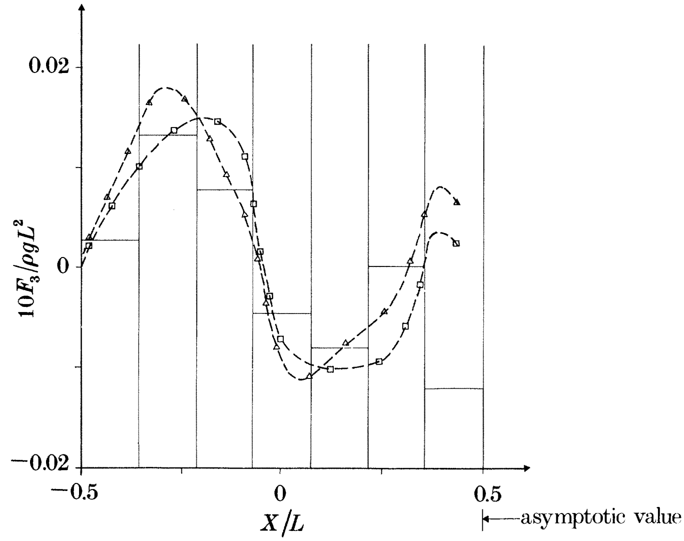

Figure 5. Measured and computed steady vertical force distribution along the model presented in figure 3 and table 1. ——, Experiments, Keuning (1988); —△—, nonlinear theory; —□—, linear theory; F_3 is the vertical force per unit length. (Buoyancy force in calm water excluded), L is the ship length, X is the longitudinal coordinate, $X = -\frac{1}{2}L$ at the bow, ρ is the mass density of water. $Fn = 1.14$. Trim is 1.62°.

code. In figure 4 is shown a comparison between the measured and computed wave profile along the ship in the steady flow problem at $Fn = 1.14$. According to Keuning the experimental values represents the local solid waterline without the influence of the spray. We have also presented numerical predictions based on a linearized version of our theory. In the linear theory we satisfy the free surface conditions (2.3) and (2.4) at $z = 0$ and neglect nonlinear terms in ϕ_s and ζ_s. We note the measured maximum wave elevation is about 70% of the draught, i.e. the order of magnitude of the transverse dimensions of the ship. This is consistent with our theoretical assumptions. The nonlinear theory predicts very well the maximum wave elevation η_{max} while the linear theory underpredicts η_{max}. At the bow of the ship the experiments show a finite value of the wave elevation while the theory starts with zero amplitude. Also at the stern there is differences between theory and experiments.

Figure 5 shows the steady vertical force per unit length along the ship in the same steady flow condition as in figure 4. The ship was divided into seven segments. The experimental values are averaged values per unit length over segments. The hydrostatic force due to the pressure term '$-\rho gz$' on the body surface $z \leqslant 0$ is not included (see equation (2.20)). The agreement between theory and experiments is good except in the aft part of the ship. The nonlinear theory agrees better than the linear theory. We expect that the flow leaves the transom stern tangentially in the downstream direction so that there is atmospheric pressure at the last section. This means the vertical force per unit length on the stern of the ship is opposite to the vertical force due to the pressure term '$-\rho gz$'. This asymptotic value is shown in the figure. Our theory is not able to predict this value.

Figure 6 shows the heave added mass distribution along the ship for the non-dimensionalized frequencies $\omega\sqrt{(L/g)} = 2.26$, 4.97 and 6.77. The Froude number is

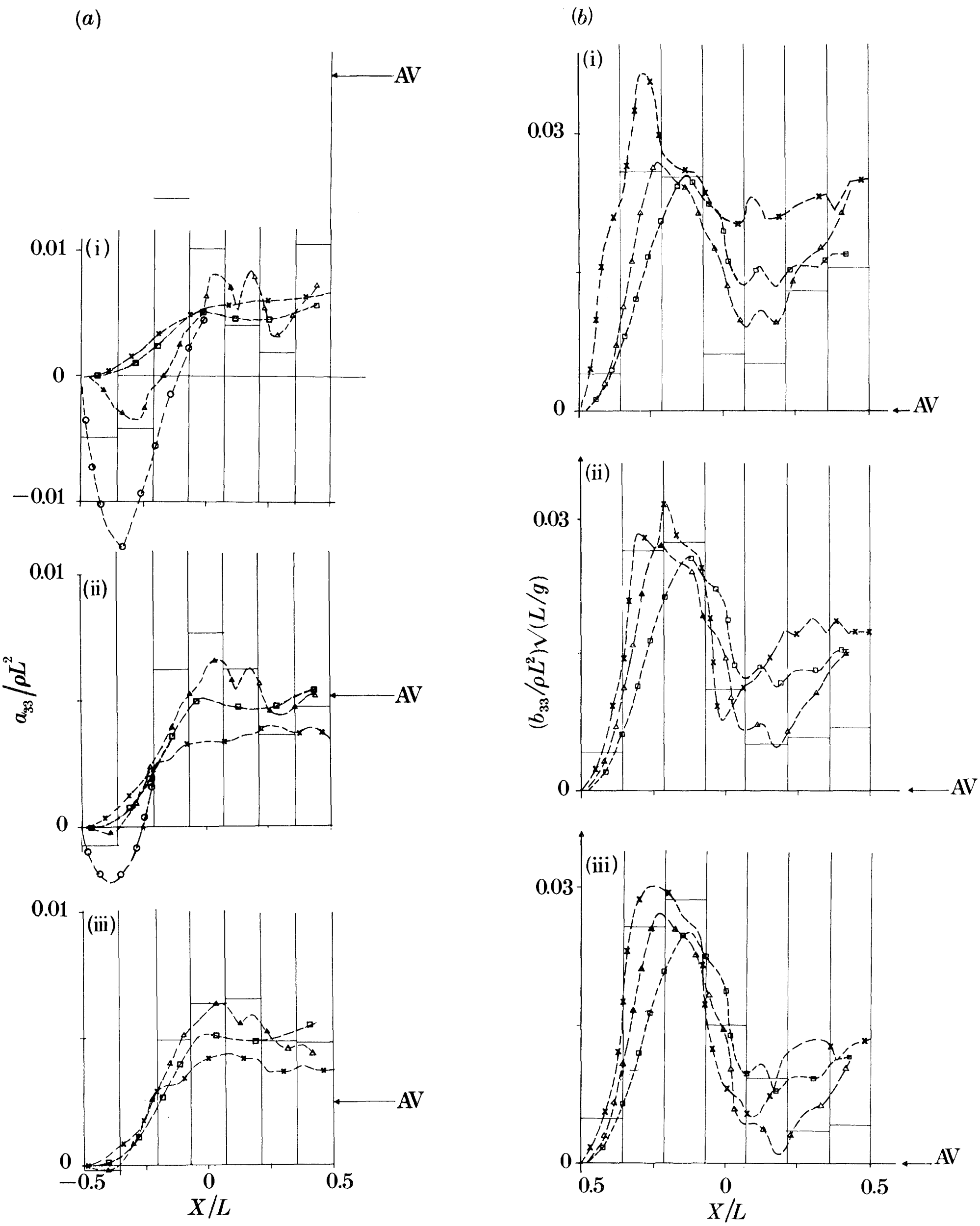

Figure 6. Heave added mass and damping distribution along the model presented in figure 3 and table 1. (i) $\omega\surd(L/g) = 2.26$; (ii) $\omega\surd(L/g) = 4.97$; (iii) $\omega\surd(L/g) = 6.77$. ——, Experiments, Keuning (1988); —△—, 'nonlinear' theory; —□—, linear theory; —×—, strip theory version 1; —○—, strip theory version 2; a_{33} is the heave added mass per unit length, b_{33} is the heave damping per unit length, ω is the circular frequency of oscillation, L is the ship length, X is the longitudinal coordinate, $X = -\frac{1}{2}L$ at the bow, ρ is the mass density of water, g is the acceleration of gravity. $Fn = 1.14$. Trim is 1.62°. AV is asymptotic value.

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1.14. The restoring coefficients are in this context defined as hydrostatic restoring coefficients in calm water, i.e. the effect of the steady wave elevation is not included in the restoring coefficients. The experimental values are presented as average values per unit length over the segments. The agreement between theory and experiments is quite good except for the third segment from the bow. The experiments show a very strong variation in added mass from segment 2 to 3. The difference between theory and experiments is more pronounced the lower the frequency is. The reason is probably stronger influence from the local steady flow than we predict.

An important part of the interaction with the local steady flow is through the m_j-terms in the body boundary conditions. These terms get very high values at the intersection between the free surface and the ship hull in the forward part of the ship. The reason is that the ship hull is not vertically wall-sided there and the second derivatives of ϕ_s become large. This indicates a singular behaviour of the m_3-term at the intersection between the free surface and the hull surface. This is likely to cause numerical errors in our predictions. The m_j-terms have larger influence the smaller the frequency is.

At the last section we have presented an asymptotic value. This is based on that the sum of the vertical dynamic force in phase with the heave displacement and the acceleration should be zero. This means the sum of what we call added mass force per unit length and restoring force per unit length should go to zero at the last section. The 'nonlinear' theory is able to predict the occurrence of negative added mass in the bow region. In general the 'nonlinear' theory shows better agreement than linear theory and strip theory. By 'nonlinear' theory we mean that the steady flow has been calculated by nonlinear theory. By linear theory we mean that we neglect all interaction terms with the steady flow problem. There are shown results from two strip theories in figure 6. Version 1 is the ordinary strip method, while version 2 is similar to the Salvesen–Tuck–Faltinsen method (1970). The strip theory calculations were done by Keuning (1988).

Figure 6 also shows the heave damping distribution along the ship. The 'nonlinear' theory agrees best with the experiments. The agreement is good except for the aft end of the ship. Our theory is not able to predict that the heave damping coefficient per unit length should be zero at the last section.

4. Conclusions

A simplified theory for ship motions at high forward speed is presented. The effect of the local steady flow is included in the body boundary conditions, free surface conditions and the pressure calculations. The local steady flow is solved by a nonlinear theory. Partly satisfactory agreement with experimental results for steady wave elevation, vertical steady force distribution and heave added mass and damping is presented. However, it is pointed out that numerical problems occur due to the high values of the m_j-terms at the intersection between the free surface and the hull surface. The reason is that the hull surface is not vertically wall-sided at the free surface. This requires further studies.

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Discussion

N. Umeda (*National Research Institute of Fisheries Engineering, Japan*). For a ship with a transom stern, the slender-body assumption fails at the aft end of the ship. However, when we consider a void space after the transom, this assumption may be still valid. Thus Professor Faltinsen's theory involuntarily deals with the slender body consisting of both the ship and the void space for vertical motions. For lateral motions, it may deal with a slender body consisting of both the ship and vortex sheets.

O. Faltinsen. I do not think it is straightforward to modify our analysis and deal with the transom stern in the way that Professor Umeda suggests. According to our theory the hydrodynamic behaviour at a cross section of the ship is independent of the downstream flow. However, what is physically happening at the transom stern will depend on the downstream flow. At the last section there will be atmospheric pressure. This is due to the void space after the transom stern. This will not be predicted by our analysis. I agree about the lateral motions when the hull ends in a trailing edge. The upstream effect of the trailing vortex sheet can be neglected. What is important is to account for the velocity jump across the vortex sheet at the trailing edge. It is believed that this can be predicted by our method.